Chapter 13

Ten (Or So) Special Formulas

In This Chapter

- Counting the number of items or groups or arrangements
- Adding up large lists of numbers
- Figuring interest earned or interest paid

A formula is actually an equation that expresses some relationship that always holds true. In this chapter, you find ten (or so) formulas that are found frequently in algebra and some mathematical studies that use a lot of algebra.

Using Multiplication to Add

The *multiplication property of counting* states that if you choose one item from the first set of choices, one item from the second set of choices, one item from the third set of choices, and so on, all you need do to find the total number of arrangements you might create is to multiply how many items are in each set.

So, if you have ten shirts, six pairs of slacks, eight pairs of socks, and three pairs of shoes, you can determine the total number of different outfits possible. Just multiply $10 \cdot 6 \cdot 8 \cdot 3 = 1{,}440$. You won't have to repeat an outfit for several years!

Factoring in Factorial

The *factorial* operation says that you take a whole number and multiply it times every natural number smaller than that whole number: $n! = n \cdot (n-1) \cdot (n-2) \ldots 3 \cdot 2 \cdot 1$. Also, by special designation, $0! = 1$.

To find 6!, you multiply $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

Picking Out Permutations

A *permutation* is a way of counting how many different arrangements are possible if you choose *r* items out of a possible *n* items and need them in a particular order:

$$_nP_r = \frac{n!}{(n-r)!}$$

So, if you have five finalists in a race and want to determine how many different ways you can have first and second place happen, you find the number of permutations possible with 5 choose 2:

$$_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 5 \cdot 4 = 20$$

The computation gives you the *number* of arrangements. Now you have to list them: Andy and Bob, Andy and Chuck, and so on.

Collecting Combinations

A *combination* is a way of counting how many different arrangements are possible if you choose *r* items out of a possible *n* items where the order they're in doesn't matter:

$$_nC_r = \frac{n!}{r!(n-r)!}$$

As you may have noticed, the only difference between permutations and combinations is that the denominator in the formula for combinations has the additional factor — making the denominator a larger number (if r isn't 0 or 1).

Adding n Integers

When you want to add $1 + 2 + 3 + 4 + \ldots + n$, use the following formula:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

So, if the bottom row of your stack has 20 blocks, the next row up has 19 blocks, and so on, the total number of blocks in your stack is

$$\sum_{i=1}^{20} i = \frac{20(20+1)}{2} = 210 \text{ blocks}$$

Adding n Squared Integers

When you want to add $1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2$, use the following formula:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

To find the sum of the first ten squares, $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 10^2$:

$$\sum_{i=1}^{10} i^2 = \frac{10(10+1)\left[2(10)+1\right]}{6} = 385$$

Adding Odd Numbers

When you want to add $1 + 3 + 5 + 7 + \ldots + (2n - 1)$, use the following formula:

$$\sum_{i=1}^{n} (2i-1) = n^2$$

Computing the sum of the first ten odd numbers, 1 + 3 + 5 + 7 + . . . + 19:

$$\sum_{i=1}^{10}(2i-1)=10^2=100$$

Going for the Geometric

A geometric sequence is formed by multiplying by the same number repeatedly. For example, multiplying by the number three, over and over again: 1, 3, 9, 27, 81, 243, ... To add up all the terms in a geometric sequence, you use the formula:

$\sum_{i=1}^{n} ar^{i-1}=\frac{a(1-r^n)}{1-r}$, where a is the first term in the sequence and r is the ratio or repeating multiplier.

So the sum of 1, 3, 9, 27, 81, and 243 is

$$\begin{aligned}\sum_{i=1}^{6} 1\cdot 3^{i-1} &= \frac{1(1-3^6)}{1-3}\\ &= \frac{-728}{-2}\\ &= 364\end{aligned}$$

And, to add to the excitement, here's the formula for finding the sum of an *infinite* geometric sequence — all the terms forever and ever:

$$\sum_{i=1}^{\infty} ar^{i-1}=\frac{a}{1-r}$$

The formula only works if the multiplier, r, is between –1 and 1.

Calculating Compound Interest

You deposit $10,000 in an account that earns 2 percent interest, compounded quarterly. How much will there be in the account at the end of 20 years? Use the following formula:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the total amount accumulated, P is the principal or beginning amount, r is the interest rate written as a decimal, n is the number of times compounded per year, and t is the number of years

So, to answer the opening question, you'd have

$$A = 10{,}000\left(1 + \frac{0.02}{4}\right)^{4(20)} \approx \$14{,}903.39$$